## 1.3 Finite Summations

A common task is the summation of many numerical values, for example in the form of elements of a set. In this section we will introduce a compact notation for this and present some standard techniques for calculating sums efficiently.

We use the sign Σ to represent summations. This is the Greek capital letter sigma. In the context of summations, this symbol is spoken as “sum.”

You may often want to sum up elements with certain properties, e.g., elements of a set, or numerical values from a certain range of numbers. Depending on what you want to express with the sum, there are different ways of writing it. The most common examples will be briefly introduced below.

One way of summing up elements is to give these elements an index, called the index of summation, and then to make the sum over all elements with a certain index. Usually, a ten below and above the sum sign. For example, to sum up the numbers the value The index is written as a subscript to the elements, while the start and end values are writ-This is read as “sum of z1 + z2 + . + zz with n, you could write the following:∑i = 1nzi z1, z2, ..., zn, i.e., start value (lower limit) and an end value (upper limit) for the index are specified for this.

Occasionally, for reasons of space, start and end values are also written to the right of thei i from 1 to n.” summation symbol instead of below and above, e.g., i = 1∑n zi. The meaning is the same.

Alternatively, you can combine the start and end value to a condition for the index and write the following equivalent to the above expression:∑1 ≤ i ≤ nzi

As you can see, sums in these notations have a certain similarity with the for-loops known from computer science.

usual in mathematics, we can choose the letters for the indices at will. The letters i, j, k, l, m, and n are often used for this purpose. For the start and end value we can use variables as well as concrete numerical values. As

Alternatively, we can write a sum without specifying the start and end value by summing

sum up all elements start and end values, we specify a condition for the elements in this case. For example, toa of a finite set A, we could write the following: up elements with a certain property, e.g., elements of a certain (partial) set. Instead of We can also combine the specification of a condition with a start and end value. For exam-∑a ∈ Aa

to sum up all even numbers between 0 and 100.ple, we could write ∑k is evenk = 0 100k

Further examplesTo add up We can also specify a concrete number for the final value, e.g., to add up 7 five times:n times the number 7 we could write the following:i = 1∑n 7 = 7+7+…+7 = n·7

Instead of a fixed number, we can also use the index itself within the sum. For example, toj = 1∑5 7 = 7+7+7+7+7 = 5·7 = 35

The start value of the summation does not always have to start at 1. We can specify anysum up all natural numbers between 1 and 10 we can write the following:n = 1∑10 n = 1+2+3+4+5+6+7+8+9+10 = 55

other numbers or variables for this purpose, as long as the start value is always smaller than the end value. With the following formula, for example, we can sum up the natural

numbers between 5 and 8:With the following formula we could sum up all odd natural numbers between 1 and 10:k = 5∑8 k = 5+6+7+8 = 26 following:To multiply and sum all elements is oddi = 1∑10 i = 1+3+5+7+9 = 25a1, ..., an by their respective index, we could write the i = 1∑n i·ai = 1·a1 +2·a2 +⋯+n·an

i − 1In order to multiply and sum the elements could write the following (note that in this case we have to start with Summation with Several Indexes correctly!) i = 2∑n ai ·ai−1 = a2 ·a1a+1, ..., a3 ·aan2 with their respective predecessors, we+⋯+an ·an−1i = 2 in order to form

We can also add up sums and thus obtain double sums, i.e., sums of sums, with two indices:

1n

Instead of often abbreviated to i = 1∑m j = 1∑n aij